$$= \lim_{x\to+\infty} x\,(x - 3\frac{\ln x}{x}) - \ln(1+\frac{4}{x^3}) = 0$$

car $\lim\limits_{x\to+\infty} \dfrac{\ln x}{x} = 0$ et $\lim\limits_{x\to+\infty} \ln(1+\dfrac{4}{x^3}) = 0$.

3°/ $\lim\limits_{x\to+\infty} x \quad \ln(\dfrac{x+1}{x}) = \lim\limits_{x\to+\infty} x \quad \ln(1+\dfrac{1}{x})$

$$= \lim_{x\to+\infty} \frac{\ln(1+\frac{1}{x})}{\frac{1}{x}} = \lim_{X\to 0} \frac{\ln(1+X)}{X} = 1 \quad (X = \frac{1}{x}).$$

4°/ $$\lim_{x\to+\infty} \frac{\ln(1+\ln x)}{x} = \lim_{x\to+\infty} \frac{\ln(\ln x\,(1+\frac{1}{\ln x}))}{x}$$

$$= \lim_{x\to+\infty} \frac{\ln(\ln x)}{x} + \frac{\ln(1+\frac{1}{\ln x})}{x}$$

$$= \lim_{x\to+\infty} \frac{\ln(\ln x)}{\ln x}\cdot\frac{\ln x}{x} + \frac{\ln(1+\frac{1}{\ln x})}{x}$$

- $\lim\limits_{x\to+\infty} \dfrac{\ln(\ln x)}{\ln x} = \lim\limits_{X\to+\infty} \dfrac{\ln X}{X} = 0 \quad (X = \ln x)$; $\lim\limits_{x\to+\infty} \dfrac{\mathrm{Log}\, x}{x} = 0$

- $\lim\limits_{x\to+\infty} \dfrac{\ln(1+\frac{1}{\ln x})}{x} = \lim\limits_{x\to+\infty} \dfrac{1}{x}\ln(1+\dfrac{1}{\ln x}) = 0\cdot 0 = 0$

d'où $\lim\limits_{x\to+\infty} \dfrac{\ln(1+\ln x)}{x} = 0$.

5°/ $\lim\limits_{x\to+\infty} x \quad \ln(\dfrac{2x+1}{2x+3}) = \lim\limits_{x\to+\infty} x \quad \ln(\dfrac{2x+3-2}{2x+3})$

$$= \lim_{x\to+\infty} x \quad \ln(1-\frac{2}{2x+3}) = \lim_{x\to+\infty} \frac{-2x}{2x+3}\cdot\frac{\ln(1-\frac{2}{2x+3})}{\frac{-2}{2x+3}}$$